

- 注意事項：
1. 答案一律寫在試卷上，否則不予計分。
 2. 請標明題號依序作答，不必抄題。
 3. 試題應隨同試卷繳回，不得攜出試場。

1. Suppose $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = \int_1^x t^2 \sqrt{3+t^4} dt + 2$.
 - (i) Determine where f is concave up or concave down. (4%)
 - (ii) Show that f has the inverse function f^{-1} . (2%)
 - (iii) Determine the range of f . (4%)
 - (iv) Determine the equation of the tangent line to the graph of f^{-1} at $(2, f^{-1}(2))$. (6%)
 - (v) Show that $\frac{4}{3} < f(0) < 2$. (4%)
2. Evaluate the following limits.
 - (i) $\lim_{x \rightarrow \infty} \frac{x - \sin x}{x}$. (5%)
 - (ii) $\lim_{k \rightarrow \infty} \sum_{i=1}^k \frac{1}{\sqrt{k^2+i}}$. (5%)
3. (i) For any $x \in \mathbb{R}$, find the sum of the series $\sum_{k=0}^{\infty} \frac{x^2}{(1+x^2)^k}$. (5%)
 - (ii) Is the improper integral $\int_0^1 \frac{\ln x}{1-x} dx$ convergent? (10%)
4. Evaluate the following integrals.
 - (i) $\int_0^1 \frac{1}{1+e^x} dx$. (7%)
 - (ii) $\iint_{\Omega} (x+y)^2 d(x,y)$, where Ω is the parallelogram bounded by the lines $x+y=0$, $x+y=1$, $2x-y=0$ and $2x-y=3$. (8%)
5. A curve C in the plane is described by $\vec{\alpha}: [0, \pi] \rightarrow \mathbb{R}^2: \vec{\alpha}(t) = (-3 \cos t, 2 \sin t)$.
 - (i) Find the area of the region enclosed by the curve C and the x -axis. (8%)
 - (ii) Find the work done by the force field $F(x, y) = (y, -x)$ in moving an object from $(-3, 0)$ to $(3, 0)$ along C . (7%)
6. Let S be the circular paraboloid $x^2 + y^2 - z = 1$. Use the gradient to find
 - (i) the direction in which z increases most rapidly at $(1, 2)$ and this maximum rate of increase, and (10%)
 - (ii) the parametric equations of the normal line to S at $(1, 2, 4)$. (5%)
7. Suppose $x, y \in (0, 1)$ and satisfy $x + y = 1$. Use the technique of Calculus to show that $2^x + 2^y < 3$. (10%)