

臺灣綜合大學系統

108 學年度 學士班

轉學生聯合招生考試

試 題

類組：A06/A07/A09/A10/A11
/B12/D26

科目名稱：微積分 A

科目代碼：E0011

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| 科目名稱 | 微積分 A | 類組代碼 | 共同考科 |
| | | 科目碼 | E0011 |

※本項考試依簡章規定各考科均「不可以」使用計算機

本科試題共計 2 頁

Answer without complete work shown receives no credit. 所有計算過程都必須詳細列出，否則不予計分。

1. (10 points) Find the following limits.

(a)

$$\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3}$$

(b)

$$\lim_{x \rightarrow 0} \frac{x^2}{\sec x - 1}$$

2. (10 points) Evaluate $\frac{\partial f}{\partial x}|_{(0,0)}$ and $\frac{\partial f}{\partial y}|_{(0,0)}$ for

$$f(x, y) = \begin{cases} \frac{3x^4 + xy^2}{2x^3 + 4xy + y} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0). \end{cases}$$

3. (10 points) Given

$$f(t) = \begin{cases} 1; & t \leq 0 \\ 1 - t; & t > 0 \end{cases}$$

and

$$F(x) = \int_{-1}^{2ax+2} f(t) dt$$

with $a > 0$, find a so that F is maximum at $x = -2a$.

4. (10 points) Find the largest possible area of a triangle with vertices $(0, 2)$, $(1, 0)$ and the third vertex on the ellipse

$$x^2 + \frac{y^2}{4} = 1.$$

5. (10 points) Evaluate

$$\int_{\ln \frac{1}{4}}^{\ln \frac{1}{2}} \frac{e^x}{\sqrt{1 - 4e^{2x}}} dx.$$

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6. (10 points) Evaluate

$$\int_0^{\frac{\sqrt{2}}{2}} \int_y^{\sqrt{1-y^2}} e^{x^2+y^2} dx dy.$$

7. (10 points) Derive the complete Taylor series expansion for

$$\ln \left(\frac{1+2x}{1-2x} \right)$$

about $x = 0$. (In the form $\sum_{k=1}^{\infty} a_k x^{2k-1}$ with a general formula for a_k .)

8. (10 points) Given function $T(x, y) = 1 + x^2 - y^2$, find the curve $\gamma(t) = (x(t), y(t))$ so that $\gamma(0) = (1, 4)$ and $\gamma'(t) = -\nabla T(\gamma(t))$.

9. (10 points) Find (a, b) with $-\frac{1}{2} \leq b \leq \frac{1}{2}$ so that

- The point $P = (1, a, b)$ is on the surface E defined by

$$\frac{x}{2} - \frac{y}{4} + \frac{\sin(2z)}{4} = 0$$

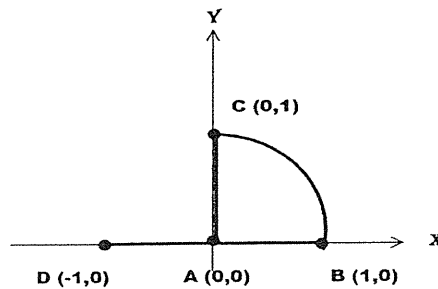
- The tangent plane to E at P contains lines

$$l_1(t) = P + t \left(\frac{1}{2}, 1, 0 \right) \quad \text{and} \quad l_2(t) = P + t(0, 2, 1)$$

10. (10 points) Evaluate $\int_L \vec{F} \cdot d\vec{r}$, where

$$\vec{F} = (4x + 5y, e^{\cos y} + 7x)$$

and L is the path from A to B , to C to A and to D as shown below:



Note: The path from B to C is circular.