系所組別：理、工、電資學院學系

考試科目：微積分(A)　　　　　　　　　　　　　　　　　　　考試日期：0714，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10 %) Find the limit : $\lim_{x \to 0} \left(\dfrac{1}{x \cdot \arcsin x} - \dfrac{1}{x^2} \right)$.

2. (10 %) Let $f(x) = \displaystyle\int_1^x x \cdot \arctan(t^2)\, dt$. Find $f'(1)$ and $f''(1)$.

3. (10 %) Evaluate $\displaystyle\int_0^\infty \dfrac{dx}{(x+4)\sqrt{x}}$.

4. (10 %) Solve the differential equation:

$$y' = (1+x)^{-1} + \sec^2 x, \quad y(0) = 1, y'(0) = 0.$$

5. (12 %) For $\displaystyle\sum_{n=1}^\infty \dfrac{3n}{n!} x^{3n-1}$, determine the interval of convergence and also find its sum.

6. (12 %) Let $f(x) = x^4 + ax^3 + b$, and have a local exterme value of -17 at $x = 3$.
   (a) Fine the value of a and b.
   (b) Is that local extreme value maximal or minimal ?
   (c) Is there any inflection point of the graph of f ?

7. (12 %) Two particles are free to move on the curves $y = x^2$ and $x - y = 1$, respectively. What are their positions when they are closest together ?

8. (12 %) Determine the values of a and b such that the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ contains the circle $(x - 1)^2 + y^2 = 1$ and has least area .

9. (12 %) Evaluate $\displaystyle\iint_R \left(\dfrac{y-x}{y+x} \right)^{1/2} dA$, where R is the trapezoid in the xy plane bounded by the lines $x = 0, y = x, x + y = 1$ and $x + y = 2$.

（本試題結束）