臺灣綜合大學系統105學年度學士班轉學生聯合招生考試試題

科目名稱	微積分A	類組代碼	E00
		科目碼	E0011
※本項考試依簡章規定各考科均「不可以」使用計算機		本試題共計	1 頁

※ 請於答案卷作答，題號並請標示清楚！

填充題：每格 5 分，8 格總共 40 分

1. $\lim_{x\to 0}\dfrac{x^2+\sin^2 x}{1-\cos x} =$ ____1(a)____ ; $\lim_{x\to 0^+}(e^x+x)^{\frac{1}{x}} =$ ____1(b)____

2. Let $x^2+xy+2y^2=1$.
   Find $\dfrac{dy}{dx} =$ ____2(a)____ and $\dfrac{d^2y}{dx^2} =$ ____2(b)____ at $(x,y)=(1,0)$.

3. Find the interval = ____3(a)____ of convergence of the power series $\sum_{n=0}^{\infty}(-1)^n\cdot\dfrac{x^{2n+1}}{2n+1}$, and its sum = ____3(b)____

4. The graph of the equation $y=\dfrac{x}{(x+3)^2}$ is strictly increasing on the interval = ____4(a)____ and concave upward on the interval = ____4(b)____.

計算與證明題：每題 10 分，6 題總共 60 分

5. Evaluate $\int_0^{\infty} x^2 e^{-x^2}dx$ from the known integral $\int_0^{\infty} e^{-x^2}dx = \dfrac{\sqrt{\pi}}{2}$

6. Show that $F(x)=\dfrac{x}{2}\sqrt{a^2-x^2}+\dfrac{a^2}{2}\arcsin\left(\dfrac{x}{a}\right)$, $a>0$ is an anti-derivative for $f(x)=\sqrt{a^2-x^2}$.

7. Let $z=\ln\left(\dfrac{1+x}{1+y}\right)$ where $x=\cos t,\ y=\tan t$. Use the chain rule to find the value of $\dfrac{dz}{dt}$ when $t=0$.

8. An observer at $(3,6)$ is watching an object descend the graph $y^2=x$. At what point of it's path is the object closest to the observer.

9. Find the area of surface $\int_S\int z^2 ds$ over the hemisphere $z=\sqrt{1-x^2-y^2}$.

10. Find the volume of the solid bounded above by the cone $z=2-\sqrt{x^2+y^2}$ and below by the disk $R: x^2+(y-1)^2\le 1$.