

- 注意事項： 1. 答案一律寫在試卷上，否則不予計分。
2. 請標明題號依序作答，不必抄題。
3. 試題應隨同試卷繳回，不得攜出試場。

1. A function f is defined by $f(x) = \lim_{n \rightarrow \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$. Where is f continuous? (10%)

2. (a) Suppose f is an even function and is differentiable at 0. Prove that $f'(0) = 0$. (8%)

(b) Suppose f is an odd function and differentiable everywhere.

Prove that for every positive number a , there exists a number c in $(-a, a)$ such that $f'(c) = \frac{f(a)}{a}$. (8%)

3. Let $f(x) = \begin{cases} -x^3 - x^2 + x + 1 & \text{for } x \leq 0 \\ e^{-x} & \text{for } x > 0 \end{cases}$.
Find the local maximum and minimum values of f . (10%)

4. (a) Evaluate the improper integral $\int_0^{\infty} x e^{-x} dx$. (8%)

(b) Show that there exists a positive number c such that

$$\int_0^c x e^{-x} dx = \frac{1}{2} \int_0^{\infty} x e^{-x} dx. \quad (8\%)$$

5. If $f(x) = \sin(x^3)$, find $f^{(99)}(0)$ and $f^{(100)}(0)$. (10%)

6. Find the points on the sphere $x^2 + y^2 + z^2 = 1$ where the tangent plane is parallel to the plane $2x + y - 3z = 3$. (10%)

7. Let (a, b, c) be a fixed point in the first octant. Find the plane through this point that cuts off from the first octant the tetrahedron of minimum volume, and determine the resulting volume. (12%)

Hint: Let the plane be $\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1$.

8. (a) Use polar coordinate to evaluate

$$\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \frac{1}{1+x^2+y^2} dy dx. \quad (8\%)$$

(b) Evaluate the line integral $\int_C x^3 y dx - x dy$ where C is the circle $x^2 + y^2 = 1$ with counterclockwise orientation. (8%)