

系所組別： 全校

考試科目： 微積分

考試日期： 0710 · 節次： 3

※ 考生請注意：本試題 可 不可 使用計算機

1. Suppose $f: (0, \infty) \rightarrow \mathbb{R}$ such that $f(x) = \frac{\ln x}{1 + \ln^2 x}$.
- (a) Find the asymptotic lines of the graph $y = f(x)$, if exist. (6%)
- (b) Find the absolute extreme values of f , if exist. (8%)
2. Define the function $E(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.
- (a) Show that $E(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)}$. (8%)
- (b) What is the domain of $E(x)$? Explain your answer. (4%)
3. Let $f(x) = \int_0^x x \cos(t^2) dt$. Find $f'(0)$ and $f''(0)$. (10%)
4. For the double integral $\int_0^1 \int_{\sin^{-1} y}^{\frac{\pi}{2}} \cos x \sqrt{1 + \cos^2 x} dx dy$,
- (a) change the order of integration to be $dy dx$; (6%)
- (b) and then evaluate the integral. (8%)
5. Let $f(x, y) = x^3 - 6xy + y^3$.
- (a) Find the critical points of $f(x, y)$. (6%)
- (b) Determine whether the critical points are points of maximum, minimum values or saddle points. (8%)
6. For a differential equation $x^2 y'' - 3xy' + 4y = 0$,
- (a) use $z = \ln x$ to transform such an equation into an equation with constant coefficients; (6%)
- (b) find the general solution of (a) in terms of x . (6%)
7. Let the function $z = f(x - y, y - x)$. Prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$. (10%)
8. Let Q be the solid region cut from the sphere $x^2 + y^2 + z^2 = 4$ by the cylinder $r = 2 \sin \theta$.
- (a) List the double integral to find the volume of Q using polar coordinate system. (6%)
- (b) Evaluate the double integral at (a). (8%)