

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (12 points) Find the following limits :

(a) (6 points)  $\lim_{x \rightarrow \infty} (\ln x)^{-2/x}$

(b) (6 points)  $\lim_{n \rightarrow \infty} \frac{1}{n} [2^{1/n} + 2^{2/n} + \dots + 2^{n/n}]$

2. (12 points) Let  $f(x) = \sin^{-1} \frac{x}{2} + \sqrt{4-x^2}$ , for each  $x \in [-2, 2]$

(a) (6 points) Find  $f'(x)$

(b) (6 points) Find extreme values of  $f(x)$

3. (10 points) Prove that :  $\frac{x-1}{x} < \ln x < x-1$ , for each  $x > 1$ .

4. (12 points) Evaluate the following integrals :

(a) (6 points)  $\int_0^{\frac{\pi}{2}} \sin 2x \cdot \cos^2 x dx$

(b) (6 points)  $\int \frac{\ln x}{\sqrt[3]{x}} dx$

5. (14 points)

(a) (6 points) Find the convergence set of  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{3n+1}$

(b) (8 points) Evaluate  $S : S = 1 - \frac{1}{4} + \frac{1}{7} - \dots + \frac{(-1)^n}{3n+1} + \dots$

6. (10 points) Define  $f(x, t) = \int_0^{x/\sqrt{2t}} e^{-s^2/2} ds$ , for  $x > 0, t > 0$ .

Prove that :  $f_t = f_{xx}$  where  $f_t = \frac{\partial f}{\partial t}, f_{xx} = \frac{\partial^2 f}{\partial x^2}$ .

7. (10 points) Determine and classify the stationary points of the function  $f(x, y) = xy(3x + 6y - 2)$ .

8. (10 points) Find the total arc length of the cardioid  $r = a(1 - \cos \theta), a > 0$ .

9. (10 points) Use change of variables  $x = u, y = \frac{v}{u}$  to evaluate the integral  $\iint_R \frac{dA}{1+x^2y^2}$ , where  $R = \{(x, y) : 1 \leq x \leq 5, 1 \leq xy \leq 5\}$ .