

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

計算題：無計算過程不給分

1. Find the following limits:

(a) (5 points) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec^2 x - 2 \tan x}{1 + \cos 4x}$

(b) (5 points) $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x+3} \right)^{x-2}$

2. Define $f(x) = \begin{cases} \frac{1}{x}[(1+x)^n - 1] & \text{if } x \neq 0 \\ A & \text{if } x = 0 \end{cases}$

(a) (5 points) Find value of A such that f is continuous at $x = 0$

(b) (5 points) Is $f(x)$ differentiable at $x = 0$? Explain.

3. Evaluate the following integrals:

(a) (6 points) $\int \sqrt{x+1} \left(x - \frac{1}{\sqrt{x}} + 1 \right) dx$

(b) (6 points) $\int_1^{\infty} \frac{\ln x}{x^4} dx$

4. (10 points) Find $\frac{du}{dt}$ at $t = \sqrt{2}$ if $u = \ln \left(\sin(xy^{-\frac{1}{2}}) \right)$, when $x = \frac{t^2}{2}$, $y = \sqrt{t^2 - 1}$.

5. (10 points) Find the area of the polar region R common to the two regions bounded by the curve $r = -6 \cos \theta$ and $r = 2 - 2 \cos \theta$

6. (10 points) Find the volume of the solid that lies under the cone $z = \sqrt{x^2 + y^2}$, about the xy-plane, and inside the cylinder $x^2 + y^2 = 2x$.

7. (10 points) If $f(x) = \ln \left(\frac{x+1}{2x+1} \right)$, find $f^{(n)}(0)$.

8. (10 points) For what value(s) of a does $\int_1^{\infty} \frac{ax}{x^2+1} - \frac{1}{2x} dx$ converge?

9. (10 points) Determine and classify the stationary points of the function $f(x, y) = x^2 + y^3 + 6xy - 7x - 6y$.

10. Convert $\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} 3r dz dr d\theta$ to

(a) (4 points) rectangular coordinates with order of integration $dz dx dy$,

(b) (4 points) spherical coordinates with order of integration $\rho d\rho d\phi d\theta$.

(Note: do not evaluate the integrals both in (a) and (b))