

1. (i) Suppose $u = f(x, y, z)$, $z = g(x, y, t)$ and $y = h(x, t)$ are differentiable real-valued functions in suitable domains. Find $\frac{\partial u}{\partial x}(x, t) = ?$
 $\frac{\partial u}{\partial t}(x, t) = ?$ 10%
- (ii) Suppose $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ is continuous and $g : \mathbf{R} \rightarrow \mathbf{R}$ is defined by $g(t) = \int_1^{t^2} \left(\int_0^{x^3} f(x, y) dy \right) dx$. Determine $g'(t) = ?$ 10%
2. (i) Is the integral $\int_{0+}^1 \sin \frac{1}{x} dx$ convergent? Justify your answer. 10%
- (ii) Let $a_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n}$. Is $\{a_n\}$ convergent? Justify your answer. 10%
3. (i) Find the work done by the force $F(x, y, z) = (yz, xz, xy)$ in moving an object from $(0, 0, 0)$ to $(1, 2, 3)$ along the curve $\vec{\gamma}(t) = (t, 2t, 3t)$. 10%
- (ii) Use Green's Theorem to evaluate $\int_C x^2 y dx + 3xy dy$, where C is the positively oriented simple closed curve determined by the graphs of $y = x^2$ and $y = \sqrt{x}$. 10%
4. Let f be differentiable for $x > 0$. Prove or disprove
- (i) If $\lim_{x \rightarrow \infty} f(x) = 0$, then $\lim_{x \rightarrow \infty} f'(x) = 0$. 10%
- (ii) If $\lim_{x \rightarrow 0+} f(x) = \infty$, then $\lim_{x \rightarrow 0+} f'(x) = -\infty$. 10%
5. (i) Evaluate $\int_0^1 \left(\int_x^1 \frac{\sin y}{y} dy \right) dx$. 10%
- (ii) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + (\tan x)^{\sqrt{3}}}$. 10%