

臺灣綜合大學系統

108 學年度 學士班

轉學生聯合招生考試

試 題

類組：C02/C11/D24

科目名稱：微積分 C

科目代碼：E0013

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科目名稱	微積分 C	類組代碼	共同考科
		科目碼	E0013
※本項考試依簡章規定各考科均「不可以」使用計算機		本科試題共計 2 頁	
<p>*** Answer without complete work shown receives no credit. No electronic devices allowed. ***</p>			
<p>1. (10 points) Find the following limits:</p>			
<p>(a) (5 points) $\lim_{x \rightarrow 0} \frac{2^x - 1}{4^x - 1}$.</p>			
<p>(b) (5 points) $\lim_{x \rightarrow 0} \sin^{-1}\left(\frac{1-x}{1-x^2}\right)$, where \sin^{-1} is the inverse function of sine.</p>			
<p>2. (10 points) Define $f(x) = \tan^2(x)$ for $x \in (0, \frac{\pi}{2})$ and let f^{-1} be its inverse function. Find $(f^{-1})'(3)$.</p>			
<p>3. (10 points) Compute the following integrals</p>			
<p>(a) (5 points) $\int_0^{\pi/2} \frac{\sin 2x}{2 + \cos x} dx$.</p>			
<p>(b) (5 points) $\int_1^2 \frac{(\ln x)^2}{x^3} dx$.</p>			
<p>4. (10 points) Find the arc length of the curve with equation $x^{2/3} + y^{2/3} = 1$ within the region $\{(x, y) \mid x \geq 0 \text{ and } y \geq 0\}$.</p>			
<p>5. (10 points) Find the slope of the tangent line to the polar curve $r = 1 + \sin(2\theta)$ at the point specified by $\theta = \pi/3$.</p>			
<p>6. (10 points) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n(x+3)^n}{4^{n+1}}$.</p>			
<p>7. (10 points) Find the Maclaurin series of order 4 for the function $f(x) = e^{-x^2} \cos x$, i.e., approximate $f(x)$ by a polynomial $a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$.</p>			

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本科試題共計 2 頁

8. (10 points) Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.

9. (10 points) Let $F(x, y) = \frac{-2y}{x^2 + y^2} \mathbf{i} + \frac{2x}{x^2 + y^2} \mathbf{j}$ and let $D = \{(x, y) \mid x^2 + y^2 = 9\}$. Find

$$\int_{\partial D} \mathbf{F} \cdot \mathbf{T} ds,$$

where we traverse the boundary ∂D in the counterclockwise direction and \mathbf{T} is the unit tangent vector.

10. (10 points) Let $F(x, y, z) = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{3/2}}$ and let $D = \{(x, y, z) \mid \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{9} = 1\}$. Find the flux of F outward across the boundary of D .