

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

計算題：無計算過程不給分

1. Find the following limits:

(a) (6 points) $\lim_{x \rightarrow 1} (1 - x) \tan \frac{\pi x}{2}$

(b) (6 points) $\lim_{x \rightarrow \infty} \left(\frac{x-1}{x+3} \right)^{x+2}$

(c) (6 points) $\lim_{x \rightarrow 0} \frac{x^2 - S(x)\sqrt{x}}{x^2}$, where $S(x) = \int_0^{\sqrt{x}} \sin(t^2) dt$

2. Evaluate the following integrals:

(a) (6 points) $\int_0^{\frac{\pi}{4}} \sec^2 x (4 - \tan^2 x)^{\frac{1}{2}} dx$

(b) (6 points) $\int \frac{\sqrt{2+x^2} - \sqrt{2-x^2}}{\sqrt{4-x^4}} dx$

3. (10 points) If $u = x^{y^2}$, find the partial derivatives $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$ at $(x, y) = (2, \sqrt{2})$.

4. (10 points) Evaluate $\int_0^{\sqrt{3}} \int_{\frac{y}{\sqrt{3}}}^{\sqrt{4-y^2}} e^{-x^2-y^2} dx dy$

5. (10 points) Find the derivative of the function $f(x) = \tan^{-1} \left(\frac{x}{a} \right) + \ln \sqrt{\frac{x-a}{x+a}}$ simplify where possible.

6. (10 points) Find the area of the polar region R, where R is bounded by the smaller loop of the curve $r = 1 + 2 \cos \theta$.

7. (10 points) Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$. Then identify the values of x for which the series converges absolutely or conditionally.

8. (10 points) In which direction from the point $(1, 1)$ is the directional derivative of $f(x, y) = x^2 + y^2$ zero?

9. (10 points) Determine and classify the stationary points of the function $f(x, y) = x^2 + y^3 + 6xy - 7x - 6y$.