

臺灣綜合大學系統 105 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 B	類組代碼	E00
		科目碼	E0012
※本項考試依簡章規定各考科均「不可以」使用計算機		本試題共計	1 頁

※ 注意：請於答案卷作答並提供完整計算過程，否則不予計分。

1. Let $f(x) = x^2 \left[1 - \cos\left(\frac{3}{x}\right) \right]$, $x \neq 0$.

1-1.(5%) Evaluate the limit. $\lim_{x \rightarrow 0} f(x)$

1-2.(5%) Evaluate the limit. $\lim_{x \rightarrow \infty} f(x)$

1-3.(10%) Evaluate the limit. $\lim_{x \rightarrow 0} f'(x)$

2. Let $f(x) = \exp\left(1 - \frac{x^2}{8}\right)$.

2-1.(5%) Find the intervals of increases or decreases of $f(x)$.

2-2.(5%) Find the intervals of concave up or concave down of $f(x)$.

2-3.(10%) Sketch the graph of $f(x)$, $x \in [-5, 5]$. Label the points of local maximum or minimum, points of inflection and asymptotes on the graph. ($e \approx 2.71$, $\sqrt{e} \approx 1.65$.)

3. Let $f(x, y) = 8x^3 - 12xy + y^3$.

3-1.(5%) Find the tangent plane of the surface $z = f(x, y)$ at $(1, 4)$.

3-2.(5%) In what direction does $f(x, y)$ have minimum rate of change at $(1, 4)$?

3-3.(10%) Find the local maximum, local minimum and saddle points of $f(x, y)$.

4.(10%) Evaluate the indefinite integral. $\int \frac{(\ln x)^2}{x^2} dx$

5.(10%) Use third order Taylor polynomial to approximate the number $\sqrt{4.05}$.

(Put the answer in the form $\sqrt{4.05} \approx 2 \pm \frac{1}{\square} \pm \frac{1}{\square} \pm \frac{1}{\square}$.)

6.(10%) The Cobb-Douglas production function is $P = x^{\frac{1}{4}}y^{\frac{1}{2}}z^{\frac{1}{4}}$ subject to total budget constraint $3x + 4y + 5z = 240$. (x : capital, y : labor, z : resource.)

Use Lagrange multiplier to find values of x, y, z such that P is maximized.

7.(10%) Evaluate the double integral. $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{\sqrt{x}(1+y^4)} dy dx$