⑧⑦ 學年度 國立成功大學 轉學生考試 系 微積分 試題 共 1 頁 第 1 頁

注意事項：1. 答案一律寫在試卷上，否則不予計分。
2. 請標明題號依序作答，不必抄題。
3. 試題應隨同試卷繳回，不得攜出試場。

1. Evaluate the left-hand limit of the function $\frac{1}{[x^2-1]}$ at 1, i.e. $\lim_{x\to 1^-}\frac{1}{[x^2-1]}$, where $[x^2-1]$ is the Gauss function of x^2-1. (10 points)

2. Let $P(x)=a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n$ be an n-th degree polynomial such that its all the coefficients $a_0, a_1, \ldots, a_n$ are reals and satisfy the equation

$$\frac{a_n}{1}+\frac{a_{n-1}}{2}+\cdots+\frac{a_0}{n+1}=0.$$

Show that the equation $P(x)=0$ has at least one real root between 0 and 1. (10 points)

3. Find the tangent equation for the function $f(x)=\sqrt{x^4-x^3}$. (10 points)

4. Determine whether the following two improper integrals are convergent ? If an improper integral is convergent, find its limit. Otherwise, explain why the improper integral is divergent?

(a) $\int_{1^+}^{+\infty}\frac{dx}{x(x-1)}$; (b) $\int_{-1^+}^{0}\frac{dx}{(x-3)\sqrt{x+1}}$. (20 points)

5. Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{+\infty}(-1)^n\frac{x^{2n-1}}{2^n}.$$

(10 points)

6. Find the Maclaurin series of the function $\sin^{-1}x$. (10 points)

7. Find the maximum and minimum values of $f(x,y,z)=x-2y+z$, with the constraints $x^2+y^2+z^2=1$ and $x+y+z=0$. (10 points)

8. Find the volume of the solid enclosed by the cone $z=r$ and the cylinder $r=3\sin\theta$ in the first-octant. (10 points)

9. Let S be the positively directed curve of the ellipse

$$\frac{x^2}{9}+\frac{y^2}{4}=1.$$

Evaluate the line integral

$$\int_S (x^4-3y)dx+(4x+2y^2)dy.$$

(10 points)

5