

- 注意事項：1. 答案一律寫在試卷上，否則不予計分。  
2. 請標明題號依序作答，不必抄題。  
3. 試題應隨同試卷繳回，不得攜出試場。

1. Please state the fundamental theorem of calculus. [10%]
2. Find the Mclaurin series for the function  $f(x) = \arctan x$ . [10%]
3. Determine whether the improper integral  $\int_0^{\infty} \frac{\sin x}{x} dx$  converges or diverges. [10%]
4. Evaluate  $\lim_{x \rightarrow 1} \left( 3x^{\frac{2}{x-1}} + \frac{5 \sin(x-1)}{x-1} \right)$ . [10%]
5. Assume that  $f : [0, 2] \rightarrow R$  is a continuous function and  $\int_0^2 f(x) dx = 2$ . Prove that there is a  $c \in [0, 2]$  such that  $f(c) = c$ . [10%]
6. Let  $a_n > 0$  for all integer  $n$ . Assume that the series  $\sum_{n=1}^{\infty} a_n$  converges. Do the following series  $\sum_{n=1}^{\infty} \sqrt{a_n}$  and  $\sum_{n=1}^{\infty} (a_n)^2$  converge? diverge? or neither? If they are in the first two cases, please prove it. Otherwise please give examples. [10%]
7. Evaluate  $\int_0^1 \int_{2x}^2 ye^{y^3} dy dx$ . [10%]
8. Find the equation of the tangent plane for the surface  $z = xy$  at  $(1, 1, 1)$ . [10%]
9. Evaluate  $\int_R e^{\frac{x-y}{x+y}} dx dy$ , where  $R$  is the region bounded by the  $x$ -axis,  $y$ -axis, and  $x + y = 4$ . [10%]
10. Find the extreme value of the function  $f(x, y) = 2x^2 + y^2 - 4x - 2y + 1$  in the region  $S = \{(x, y) \mid \frac{x^2}{9} + \frac{y^2}{18} \leq 1\}$ . [10%]