臺灣綜合大學系統

107 學年度 學士班

轉學生聯合招生考試

試　　題

類組：A12/A14/C06/D38

科目名稱：微積分 B

科目代碼：E0012

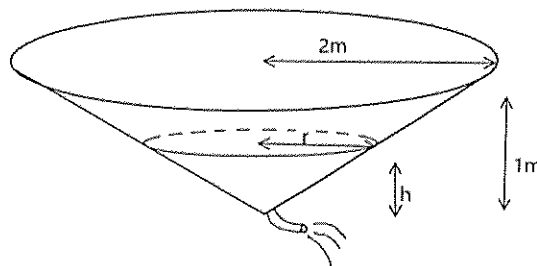| 科目名稱	微積分 B	類組代碼	共同考科
		科目碼	E0012

※本項考試依簡章規定各考科均「不可以」使用計算機　　　　本科試題共計　2　頁

Answer without complete work shown receives no credit. 所有計算過程都必須詳細列出，否則不予計分。

1. (10 points) Find the following limits.

$$\lim_{x \to 0} \frac{\sin(4x^3)}{x}.$$

2. (10 points) Consider a circular conic tank shown below. The water is drained at the bottom at the rate of $2 \ m^3$ per second. Find the rate of change of height h when the top circular surface of water has radius $1m$.

Recall that the volume of a circular cone is $\frac{1}{3}\pi r^2 h$.

3. (10 points) Evaluate the indefinite integral

$$\int \sin(\sqrt{x}) \, dx.$$

4. (10 points) Find the equation of tangent plane to the surface defined by

$$x^2 + 2y^2 + xy + e^z = 2$$

at the point $P = (1, 0, 0)$.

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5. (10 points) Evaluate the improper integral

$$\int_1^\infty \frac{3x^2+1}{x^4+x^2}\, dx.$$

6. (10 points) Consider the function

$$f(x,y) = \begin{cases} \frac{2x^3+xy}{x^2+y^2} & ;(x,y) \neq (0,0) \\ 0 & ;(x,y) = (0,0). \end{cases}$$

Find $\frac{\partial f}{\partial x}$ at $(0,0)$ if existed. If it does not exist, explain why.

7. (10 points) Find the point where local *minimum* for the function

$$f(x) = \int_0^{x^3-3x} e^{\cos(t^2+1)}\, dt$$

occurs.

8. (10 points) Write down the Taylor series expansion for the function

$$f(x) = \frac{x^3}{1+x^2}$$

at $x = 0$.

9. (10 points) Compute the integral

$$\int_0^1 \int_x^1 \cos(y^2 + 1)\, dy dx.$$

10. (10 points) Use Lagrange multiplier method to maximize the function

$$f(x,y,z) = 2x + 3y + 5z$$

on the sphere

$$x^2 + y^2 + z^2 = 19.$$

Note: any other method receives no credit.