

編號: 22 系所: 全校系所

科目: 微積分

本試題是否可以使用計算機:  可使用,  不可使用 (請命題老師勾選)

1. Show that  $\tan^{-1} \frac{x+1}{x-1} + \tan^{-1} x = C$ , where  $C$  is a constant.  
And find the value of  $C$ . (Note:  $\tan^{-1} x = \arctan x$ ) (10%)

2. Find the convergence set for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-4)^n}{n+1},$$

and also find its sum. (10%)

3. Find the average value of  $f(x) = \tan^{-1} \frac{x}{2}$  on the interval  $[0, 2]$ . (10%)

4. For  $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\tan x}{(\ln \cos x)^2} dx$ ,

(a) show that its an improper integral; (5%)

(b) evaluate or show that its diverges. (5%)

5. The position vector of a particle at time  $t \geq 0$  is

$$\vec{r}(t) = (\cos t + t \sin t)\vec{i} + (\sin t - t \cos t)\vec{j}.$$

(a) Show that the speed  $ds/dt = t$ . (5%)

(b) Show that the tangential and normal components of acceleration,  
 $a_T = 1$  and  $a_N = t$ , respectively. (5%)

6. Determine whether there are any points on the surface  $z^2 + xy - 2x - y^2 = 1$   
at which the tangent plane is parallel to  $z = 2$ . (10%)

7. The function  $f(x, y) = 6x^2 - 8x + 2y^2 - 5$  is continuous on the closed region  
 $R$  defined by  $x^2 + y^2 \leq 1$ . Find its absolute extrema over  $R$ . (10%)

8. The graph of  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ , is called a hypocycloid,

(a) find equations of the tangent lines to the graph at the points  
corresponding to  $x = \frac{1}{8}$ ; (5%)

(b) also find  $\frac{d^2y}{dx^2}$  for the equation in (a) for those points corresponding  
to  $x = \frac{1}{8}$ . (5%)

9. Compute  $\iint_R \frac{\cos \frac{1}{2}(x-y)}{3x+y} dA$ , where  $R$  is the region bounded by the  
graphs of  $y = x$ ,  $y = x - \pi$ ,  $y = -3x + 3$ , and  $y = -3x + 6$ . (10%)

10. Find the surface area of the portions of the sphere  $x^2 + y^2 + z^2 = 4$  that are  
within the cylinder  $(x-1)^2 + y^2 = 1$ . (10%)