

臺灣綜合大學系統

106 學年度

轉學生聯合招生考試

試 題

類組：A07/D01

科目名稱：線性代數

科目代碼：A0702

臺灣綜合大學系統 106 學年度學士班轉學生聯合招生考試試題

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※本項考試依簡章規定各考科均「不可以」使用計算機

本科試題共計 1 頁

In the following, $\mathbb{R}^{m \times n}$ denotes the class of all $m \times n$ real matrices. It is a real vector space with respect to matrix addition and scalar multiplication. Vectors in \mathbb{R}^n will be regarded as column vectors.

- Let $A \in \mathbb{R}^{n \times n}$. Show that if A is symmetric and satisfies $Ax \cdot x = 0$ for all $x \in \mathbb{R}^n$, then $A = 0$. (10 points)
 - Give an example to show that the assumption "symmetric" in (a) is necessary. (10 points)
- Let X be the linear subspace of $\mathbb{R}^{2 \times 3}$ containing all matrices whose columns add to $0 \in \mathbb{R}^2$. For example, $\begin{bmatrix} 1 & 2 & -3 \\ -2 & -2 & 4 \end{bmatrix} \in X$ since $\begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Similarly let Y be the subspace of $\mathbb{R}^{2 \times 3}$ containing all matrices whose rows add to $0 \in \mathbb{R}^3$. Answer the following questions with reasons.

 - What is the dimension of X . (10 points)
 - What is the dimension of $X + Y$. (10 points)
- Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. For $u, v \in \mathbb{R}^3$, define $\langle u, v \rangle = u^T Av$, then $\langle \cdot, \cdot \rangle$ is an inner product on \mathbb{R}^3 (no need to prove it). Under this inner product, find the orthonormal basis $\{f_1, f_2, f_3\}$ by applying the Gram-Schmidt process to $\{(1, 0, 0)^T, (0, 1, 0)^T, (0, 0, 1)^T\}$. (15 points)
- In \mathbb{R}^2 , let L be the line $y = mx$, where $m \in \mathbb{R}$. Find the matrix $A \in \mathbb{R}^{2 \times 2}$ so that $x \mapsto Ax$ is the reflection on \mathbb{R}^2 about L . (15 points)
- Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2 \times 2}$. Under the ordered basis $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ of $\mathbb{R}^{2 \times 2}$, compute the 4×4 matrix representing the linear transformation $M \mapsto AM$ (left multiplication by A) on $\mathbb{R}^{2 \times 2}$. (15 points)
- Find an $A \in \mathbb{R}^{2 \times 2}$ (other than $A = I$) such that $A^3 = I$. (15 points)