臺灣綜合大學系統

108 學年度 學士班

轉學生聯合招生考試

試　　題

類組：A07/C11

科目名稱：線性代數

科目代碼：A0702

臺灣綜合大學系統108學年度學士班轉學生聯合招生考試試題

科目名稱	線性代數	類組代碼	A07.C11
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※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計 3 頁

一、單選題（占 28 分）請於答案卷上作答，否則不予計分

說明：第 1 題至第 4 題，每題有 8 個選項，答案以英文字母大寫 ABCDEFGH 作答。各題答對者，得 7 分。答錯、未作答或多於一個答案者，該題以零分計算。

1. (7pts) A is a m-by-n matrix, and the system $A\vec{x}=\vec{0}$ has a unique solution $\vec{x}=\vec{0}$.
   (i) $\forall\vec{b}\in R^m$, there exists at least one solution to $A\vec{x}=\vec{b}$.
   (ii) If there is a solution to $A\vec{x}=\vec{b}$, then this is a unique solution.
   (iii) $m\le n$.

   Which of the above statements are correct (if any)?______.
   A. (i); B. (ii); C. (iii); D. (i)(ii); E. (i)(iii); F. (ii)(iii); G. (i)(ii)(iii); H. None.

2. (7pts) A and B are two n-by-n real matrices.
   (i) det(AB)=det(A)det(B).
   (ii) AB and BA have the same eigenvalues.
   (iii) I is the n-by-n identity matrix. If $AB=I$, then $BA=I$.

   Which of the above statements are correct (if any)?______.
   A. (i); B. (ii); C. (iii); D. (i)(ii); E. (i)(iii); F. (ii)(iii); G. (i)(ii)(iii); H. None.

3. (7pts) A and B are two similar n-by-n real matrices.
   (i) A and B have the same characteristic polynomial.
   (ii) A and B have the same eigenvectors.
   (iii) If A is symmetry, then B is symmetry.

   Which of the above statements are correct (if any)?______.
   A. (i); B. (ii); C. (iii); D. (i)(ii); E. (i)(iii); F. (ii)(iii); G. (i)(ii)(iii); H. None.

4. (7pts) Matrix A is symmetric positive definite and matrix Q is orthogonal.
   (i) Q^TAQ is a diagonal matrix.
   (ii) Q^TAQ is symmetric positive definite.
   (iii) All pivots of Q^TAQ (without row changes) are positive.

   Which of the above statements are correct (if any)?______.
   A. (i); B. (ii); C. (iii); D. (i)(ii); E. (i)(iii); F. (ii)(iii); G. (i)(ii)(iii); H. None.

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二、單選題（占 72 分）請於答案卷上作答，否則不予計分

說明：1. 第 5 題至第 14 題，每題有 9 個選項，答案以英文字母大寫 ABCDEFGHI 作答。
2. 第 5 至 12 題答對給 7 分，第 13 至 14 題答對給 8 分。答錯、未作答或多於一個答案者，該題以零分計算。

5. (7pts) Matrix $A=\begin{bmatrix}1/2 & -4 & -2\\ 0 & 2 & 2\\ 0 & 0 & -1\end{bmatrix}$. $A^{-1}=\begin{bmatrix}* & a & b\\ * & * & c\\ * & * & *\end{bmatrix}$. Then $a+b+c=$______.

A. 1; B. 2; C. 3; D. 4; E. 5; F. 6; G. 7; H. 8; I.9.

6. (7pts) Linear transformation $T: \boldsymbol{R}^2 \to \boldsymbol{R}^3$ satisfies $T\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}2\\2\\a\end{bmatrix}$ and $T\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}3\\b\\6\end{bmatrix}$. If T is ***not*** one-to-one, then $a+b=$______.

A. 1; B. 2; C. 3; D. 4; E. 5; F. 6; G. 7; H. 8; I.9.

7. (7pts) Matrix $A=\begin{bmatrix}0 & 2 & 4 & 2\\ 1 & 12 & 5 & 4\\ 2 & 22 & 6 & 6\\ 3 & 32 & 7 & 8\\ 4 & 42 & 8 & 0\end{bmatrix}$. Rank$(A)=$______.

A. 1; B. 2; C. 3; D. 4; E. 5; F. 6; G. 7; H. 8; I.9.

8. (7pts) $T\left(\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}\right)=\begin{bmatrix}-1 & -1 & 3 & 0 & 3\\ 0 & -1 & 2 & 1 & 1\\ 1 & -2 & 3 & 3 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}$ is a linear operator from $\boldsymbol{R}^5$ to $\boldsymbol{R}^3$.

$\mathrm{Ker}(T)=\left\{\alpha\begin{bmatrix}a\\d\\1\\0\\0\end{bmatrix}+\beta\begin{bmatrix}b\\e\\0\\1\\0\end{bmatrix}+\gamma\begin{bmatrix}c\\f\\0\\0\\1\end{bmatrix}, \forall \alpha,\beta,\gamma\in\boldsymbol{R}\right\}$. Then $a+c+e=$______.

A. 1; B. 2; C. 3; D. 4; E. 5; F. 6; G. 7; H. 8; I.9.

9. (7pts) Matrix $A=\begin{bmatrix}3 & 0 & 5\\ 2 & -1 & 2\\ 1 & 0 & 2\end{bmatrix}$, $A^{-1}=aA^2+bA+cI$. Then $a+b+c=$______.

A. 1; B. 2; C. 3; D. 4; E. 5; F. 6; G. 7; H. 8; I.9.

　　背面有題，請繼續作答。

10. (7pts) Given five points $(x_1, y_1) = (-2,9)$, $(x_2, y_2) = (-1,2)$, $(x_3, y_3) = (0,-6)$, $(x_4, y_4) = (1,0)$, $(x_5, y_5) = (2,0)$. The constants C, D, E are undetermined real numbers. Find the best fitting parabola $Cx^2 + Dx + E$ to the above five points, using least squares. In other words, the best solution C, D, E is the one that minimizes $\sum_{i=1}^{5}(C(x_i^2) + Dx_i + E - y_i)^2$.
Then $C - D - E =$______.
A. 1; B. 2; C. 3; D. 4; E. 5; F. 6; G. 7; H. 8; I.9.

11. (7pts) Let $x, y \in \boldsymbol{R}$. The minimum of $\frac{5x^2-2xy+5y^2}{x^2+y^2}$ is a. Then $a =$______.
A. 1; B. 2; C. 3; D. 4; E. 5; F. 6; G. 7; H. 8; I.9.

12. (7pts) P_3 is the set of all real polynomials of degree less or equal to 3 with ordered basis $\beta_P = \{x^3, x^2, x, 1\}$. M_{2x2} is the set of all real 2-by-2 matrices with ordered basis $\beta_M = \{\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\}$.
A linear operator $T: P_3 \to M_{2x2}$ is defined as $T(f(x)) = \begin{bmatrix}f(-1) & f(0)\\ f(1) & f(2)\end{bmatrix}$.
$[T]_{\beta_P}^{\beta_M} = \begin{bmatrix}a & * & * & *\\ 0 & 0 & 0 & b\\ * & * & c & *\\ * & d & * & *\end{bmatrix}$ is the matrix representation of T with respect to β_P and β_M.
Then $a + b + c + d =$______.
A. 1; B. 2; C. 3; D. 4; E. 5; F. 6; G. 7; H. 8; I.9.

13. (8pts) Matrix $A = \begin{bmatrix}9/2 & 7/2\\ 7/2 & 9/2\end{bmatrix}$, $A^{1/3} = \begin{bmatrix}a & c\\ b & d\end{bmatrix}$. Then $a + b + c + d =$______.
A. 1; B. 2; C. 3; D. 4; E. 5; F. 6; G. 7; H. 8; I.9.

14. (8pts) P_2 is the set of all real polynomials of degree less or equal to 2. Define the inner product on P_2 by $< f, g >= \int_0^1 f(x)g(x)dx, \forall f, g \in P_2$. Let S be the subspace of P_2 spanned by $\{x^2, x\}$. For $u(x) = 1$, there exists $v \in S$ and $w \in S^{\perp}$ so that $u = v + w$. Find $w(x)$ in the form $w(x) = 1 + ax + bx^2$. Then $a + 3b =$______.
A. 1; B. 2; C. 3; D. 4; E. 5; F. 6; G. 7; H. 8; I.9.