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		科目碼	A0702

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1. 單選題(1～20題,每題3分)

1. Consider the matrix
$$A = \begin{pmatrix} 2 & 7 & 1 \\ 1 & 3 & 0 \\ 1 & 4 & 0 \end{pmatrix}.$$
Which statement must be true?
(A) $\det(A) = 1$.
(B) $\det(A) = 2$.
(C) $\det(A) = 3$.
(D) $\det(A) = -1$.

2. Let $U = span\{u_1, u_2, u_3\}$, where
$$u_1 = (1, 0, 1, 0), u_2 = (1, 1, 1, 0), u_3 = (1, 1, 0, 0).$$
Let V be the orthogonal complement of U. Which statement must be true?
(A) $(1, 2, 3, 4)$ lies in V.
(B) $(1, 0, 0, 4)$ lies in V.
(C) $(0, 0, 0, -1)$ lies in V.
(D) $(1, 1, -1, -1)$ lies in V.

3. Let $M_{2,2}$ be the space consisting of all 2×2 matrices. Let T be a linear transformation, $T : M_{2,2} \to M_{2,2}$, $T(X) = XA - AX$, where
$$A = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}.$$
Let $ker(T)$ be the kernel space of T. Then the dimension of $ker(T)$ is
(A) 0
(B) 1
(C) 2
(D) 3
(E) 4.

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4. Let

$$A=\begin{pmatrix}4&2\\2&1\end{pmatrix}, B=\begin{pmatrix}4&2i\\2i&1\end{pmatrix}, C=\begin{pmatrix}4&1+2i\\1-2i&1\end{pmatrix}, D=\begin{pmatrix}4i&2i\\2i&i\end{pmatrix}.$$

Which matrices are hermitian?

(A) only A, B.
(B) only A, C.
(C) only A, D
(D) A, B, C, D.
(E) only B, C, D.

5. Let

$$A=\begin{pmatrix}1&2\\2&1\end{pmatrix}, B=\begin{pmatrix}1&-2\\-2&1\end{pmatrix}, C=\frac{1}{\sqrt{5}}\begin{pmatrix}2&1\\1&2\end{pmatrix}, D=\frac{1}{\sqrt{5}}\begin{pmatrix}2&-1\\-1&2\end{pmatrix}.$$

Which matrices are positive definite?

(A) only A, B.
(B) only A, C.
(C) only A, D
(D) A, B, C, D.
(E) only C, D.

6. Let

$$A=\begin{pmatrix}1&2\\2&-1\end{pmatrix}, B=\begin{pmatrix}1&2i\\2i&-1\end{pmatrix}, C=\frac{1}{\sqrt{5}}\begin{pmatrix}1&2i\\-2i&1\end{pmatrix}, D=\frac{1}{\sqrt{5}}\begin{pmatrix}1&2i\\2i&1\end{pmatrix}.$$

Which matrices are unitary?

(A) only A, B.
(B) only C, D.
(C) only C
(D) A, B, C, D.
(E) only D.

7. Let

$$A=\begin{pmatrix}1&2\\2&1\end{pmatrix}, B=\begin{pmatrix}1&-2\\-2&1\end{pmatrix}, C=\frac{1}{\sqrt{5}}\begin{pmatrix}2&1\\1&2\end{pmatrix}, D=\frac{1}{\sqrt{5}}\begin{pmatrix}2&-i\\i&2\end{pmatrix}.$$

Which matrices can be unitarily diagonalizable?

(A) only A, B.
(B) only A, C.
(C) only A, D
(D) A, B, C, D.
(E) only C, D.

背面有題，請繼續作答。

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8. Let S, T, U be transformations, $\mathbf{R}^3 \to \mathbf{R}^3$ with

$$S(x,y,z) = (x,y,0),\ T(x,y,z) = (x+y, x+z, |z|),\ U(x,y,z) = (y,z,1),$$

Then

(A) Only S is linear.
(B) Only S, T are linear.
(C) S, T, U are linear.
(D) Only S, U are linear.

9. Let a, b, c in $\mathbf{R}^4$, $a = (1,0,-2,5)$, $b = (2,1,0,-1)$. Which of the following statement must be correct?

(A) When $c = (1,1,2,1)$, then $\{a,b,c\}$ is independent in $\mathbf{R}^4$.
(B) When $c = (3,1,-2,4)$, then $\{a,b,c\}$ is independent in $\mathbf{R}^4$.
(C) When $c = (2,0,-4,10)$, then $\{a,b,c\}$ is independent in $\mathbf{R}^4$.
(D) When $c = (4,2,0,-2)$, then $\{a,b,c\}$ is independent in $\mathbf{R}^4$.

10. Let

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}.$$

Which of the following statements must be correct?

(A) 2 is one eigenvalue and the corresponding eigenvector is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.
(B) -1 is one eigenvalue and the corresponding eigenvector is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.
(C) 2 is one eigenvalue and the corresponding eigenvector is $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.
(D) 1 is one eigenvalue and the corresponding eigenvector is $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.
(E) -1 is one eigenvalue and the corresponding eigenvector is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

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11. In performing Gaussian Elimination on a matrix, which one of the following operations may NOT be allowed? :

(A) Interchange any two rows

(B) Multiply any row by a nonzero scalar

(C) Add any scalar multiple of a row to another

(D) Replace any row by another

12. Which of the following 1x3 matrices are in their reduced row echelon forms?

A=(0,0,0), B=(1,0,0),C=(0,1,0),D=(0,0,1),E=(1,2,3)

(A) None of them

(B) Only B,C,D

(C) Only A,B,C,D

(D) Only E

(E) All of them

13. Which of the following is NOT a solution of the following system of equations:

$$\begin{cases} x-2y-z+3u=1 \\ 2x-4y+z \quad\quad =5 \\ x-2y+2z-3u=4 \end{cases}$$

(A) (x, y, z, u) = (2, 0, 1, 0)

(B) (x, y, z, u) = (4, 1, 1, 0)

(C) (x, y, z, u) = (1, 0, 3, 1)

(D) (x, y, z, u) = (3, 1, 3, 1)

(E) (x, y, z, u) = (1, 0, 1, 0)

14. Which of the following operation on a plane is NOT a linear operator?

(A) Reflection of points with respect to the line y=2x

(B) Orthogonal projection of points onto the line y=2x

(C) Rotation clockwise by the acute angle formed by y=2x and the x-axis

(D) Translating by one unit in the direction of a directional vector of the line y=2x.*

(E) Reflection of points with respect to the original.

15. Among the four linear operators in question 14, how many of them are diagonalizable over the field of real numbers?

(A) 4

(B) 3

(C) 2

(D) 1

(E) 0

背面有題，請繼續作答。

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16. Which of the following statement is NOT correct in general for a vector space V over the field R(the set of all real numbers) of dimension 3?
(A) We can always find a subset of V containing 4 vectors that spans V.
(B) We can always find a subset of V containing 2 vectors that are linearly independent.
(C) We can always find a basis of V containing 3 vectors.
(D) We can always find a subspaces of V of dimension 0, 1, 2, or 3.
(E) We can always find the basis {(1, 0, 0), (0, 1, 0), (0, 0, 1)} for V.

17. Which of the following statement is NOT correct in general for a linear operator T on the vector space C^3 over the field C (the set of all complex numbers)?
(A) V is a direct sum of eigenspaces of T.
(B) The characteristic polynomial of T splits.
(C) The set span({T,T^2,T^3,....}) is a finite dimensional subspace of L(C^3), the vector space consisting of all linear operator on C^3.
(D) Any two matrix representations of T are similar 3x3 matrices.
(E)T(0,0,0)=(0,0,0).

18. Let R^3={(a,b,c): a, b, c are complex numbers}. Which of the following IS correct?
(A) R^2= {(a, b): a, b are real numbers} is a linear subspace of R^3
(B) R^3 with the usual vector addition and scalar multiplication on C is a vector space over C, where C is the set of all complex numbers.
(C)R^3 with the usual vector addition and scalar multiplication on Q is a vector space over Q, where Q is the set of all rational numbers.
(D)R^3 with the usual vector addition and scalar multiplication on Z is a vector space over Z, where Z is the set of all integers.
(E) There exists a unique inner product on R^3 as a vector space over R with usual vector addition and scalar multiplication.

19. Let P^2 be the vector space of all polynomial of degree $\leqq$ 2. Which of the following is NOT a basis for P^2?
(A) {1, x, x^2}
(B) {(x-1)(x-2)/(3-1)(3-2), (x-1)(x-3)/(2-1)(2-3),(x-2)(x-3)/(1-2)(1-3)}
(C) {1, 1+x, 1+x+x^2}
(D) {x+1, x+2, x+3}
(E){(x-1)(x-2), (x-1)(x-3),(x-2)(x-3)}

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20. How many among the seven conditions in below are equivalent to the condition that a square n by n matrix A is invertible?

(1) $\mathrm{Det}(A) \neq 0$.

(2) There exist a nxn matrix B such that $AB=I_n$, the identity matrix of order n.

(3) There exist a nxn matrix B such that $BA=I_n$, the identity matrix of order n.

(4) The homogeneous system $Ax=(0,0,0)^t$, where $x=(x_1,x_2,x_3)^t$ has only the trivial solution.

(5) Rank(A)=n.

(6)The reduce row echelon form of A is I_n.

(7) The system $Ax=(a, b, c)^t$, where $x=(x_1,x_2,x_3)^t$ has a unique solution for all a,b,c.

(A) 3

(B) 4

(C) 5

(D) 6

(E) 7

背面有題，請繼續作答。

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2. 多選題（21～30題，每題4分）

21. Let A be a square, real matrix. Which of the following statements must be correct?
(A) When the rows of a matrix A are an orthogonal set, then $A^{-1} = A^{\top}$.
(B) When the rows of a matrix A are an orthonormal set, then $A^{-1} = A^{\top}$.
(C) If a matrix A is symmetric, then A is orthogonally diagonalizable.
(D) If a matrix A is orthogonally diagonalizable, then A must be symmetric.

22. Let A be a square complex matrix. Which of the following statements must be correct?
(A) When A is a hermitian matrix, then A is symmetric.
(B) When A is a hermitian matrix, then A has real eigenvalues.
(C) When A is a hermitian matrix, then eigenvectors of A corresponding to distinct eigenvalues are orthogonal.
(D) When A is a hermitian matrix, then A is unitarily diagonalizable.

23. Consider the matrix

$$A = \begin{pmatrix} 2 & 7 & 1 \\ 1 & 3 & 0 \\ 1 & 4 & -1 \end{pmatrix}.$$

Let

$$A^{-1} = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}.$$

Which of the following statements must be correct?
(A) $a_{1,1} = -1.5$.
(B) $a_{1,2} = 5.5$.
(C) $a_{2,2} = 1.5$.
(D) $a_{3,2} = -1$.
(E) $a_{3,3} = -0.5$.

24. Let A be one $n \times m$ matrix with$m \geq n \geq 2$. Let r be the rank of A. Which statements must be true?
(A) When $m = n$, then A is invertible if and only if there exists no nonzero vector x satisfying $Ax = 0$.
(B) Suppose AC is one identity matrix for some $n \times m$ matrix C. Then CA must be one identity matrix.
(C) Suppose $m = n$. When the determinant of A is zero, the rank of A is n.
(D) The rank of $A^{\top}$ is r.
(E) We can find some invertible matrices U, V, such that

$$UAV = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$$

for some $r \times r$ identity matrix I_r.

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25. Let V, W be finite dimensional vector spaces. Let $T : V \to W$ be a linear transformation. Let $ker(T)$ and $im(T)$ be the kernel and image of T. Which statements must be correct?

(A) The kernel $ker(T)$ is a subspace of V.

(B) Suppose S is another linear transform from V to W. Suppose V is spanned by $\{v_1, v_2, \ldots, v_m\}$. If $S(v_i) = T(v_i)$ for $i = 1, \ldots, m$, then $S(v) = T(v)$ for every $v \in V$.

(C) The following dimension relation holds,

$$dimV = dim(ker(T)) + dim(im(T)).$$

(D) Suppose T is onto, then $ker(T) = \{0\}$.

背面有題，請繼續作答。

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26.Let A be a matrix of rank n. Which of the followings are correct?
(A) The maximal number of linearly independent rows in A is n.
(B) The maximal number of linearly independent column in A is n.
(C) A must be a square n by n matrix.
(D) The rank of L_A is n, where L_A is the left multiplication map.
(E) QAP has rank n for any invertible P and Q such that the matrix multiplication is defined.

27.Let L_A be the left multiplication map from R^n to R^m, Which of the followings are correct?
(A) A must be a nxm matrix.
(B) rank(A)$\leqq$ m
(C) rank(A)$\leqq$n
(D) If m=n, then L_A is invertible if and only if A is invertible.
(E) If m=n, then L_A is one-to one if and only if L_A is onto.

28.Let V be the vector space over R consisting of all functions from R to R having derivatives of all orders, called the set of smooth functions. T is the linear operator on V defined by T(f)=df/dx (the derivative of f).Which of the following sets of V are T-invariant subspaces?
(A) {all polynomial functions in x}
(B) {all polynomial functions in x of degree smaller or equal to 100}
(C) {all smooth functions f with df/dx=2f}
(D) {the set of all smooth functions g such that there exists a smooth function f with df/dx=g}
(E) {the set of all smooth functions f such that df/dx is the zero function}

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29. Let V be a vector space over R with dimension 2. Which of the followings are correct?

(A) V is isomorphic to R^2.

(B) The dual space V* has also dimension 2.

(C) There exist infinitelymany sets of basis for V.

(D) All bases contain only 2 vectors.

(E) Any linear operator on V has at most 2 distinct eigenvalues.

30. Let A be a n by n matrix with determinant 1. Which of the following matrices also have determinant 1?

(A) $B=A^t$, the transpose of A.

(B) $B=Q^{(-1)}AQ$, where Q is a change of coordinate matrix.

(C) $B=A^{(-1)}$

(D) B is the reduced row echelon form of A

(E) $B=I_n$, where is the identity matrix of order n.